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HW-27799

ATOMIC WEAPON DATA

CC: 1, 2 C. J. Shortess, Jr.
3. L. M. Meeker
4. C. A. Bennett
5. W. D. Peterson - R. F. Call
6. 300 Area Files
7. 700 Area Files
8. Yellow Copy

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C. J. Shortess, Jr.
Accountability Section
703 Building, 700 Area

STATISTICALLY DETERMINED PRECISION OF PLUTONIUM CONTENT OF SHAPES

1. INTRODUCTION

Measurement of the plutonium content of shapes is accomplished by weighing the shapes and multiplying the observed weight by a constant factor (currently 0.99) to correct for impurities. It is thus seen that there are two possible sources of error in these measurements:

a. Error in weighing
b. Error in the constant correction factor

The primary purpose of this paper is to report the magnitude of random errors found in these two sources.

2. SUMMARY

a. It is recommended that, in correcting shape weights for impurities, a factor of 0.9886 be used, unless so many significant digits are not needed.

b. It was found that a statement of the total weight of plutonium in n shapes has a precision (95 percent level) of $(0.10/\sqrt{n})$ percent.

c. It is recommended that provision be made for periodic re-examination of the correction factor and precision mentioned above.

d. As incidental results, the precisions of measurements of two impurities were found to be as follows:

Impurities	Standard Error Of Measurement	Precision (95% Level)
"70-58"	758.78 ppm.	1487 ppm. (or 14.7% relative to average impurity, 10130 ppm
Spectro-chemical*	279.38 ppm.	548 ppm. (or 60.4% relative to average impurity, 906.7 ppm.

* Impurities measured spectro-chemically, as follows: Al, Be, Ca, Cr, Fe, La, Li, Mg, Mn, Na, Ni, and Si.

3. WEIGHING OF SHAPES

In the weighing of shapes, one scale is used before coating and one after. Twenty pairs of measurements were made on the former and found to agree exactly to the five significant digits used in reporting weights. Twenty-five were made on the latter and found to agree exactly to the six significant digits used. Therefore, it is concluded that the error in weighing is negligible.

4. CORRECTION FOR IMPURITIES

a. Data

For August, October and December of 1952, data were taken for use from "234-5 Analytical Laboratory Daily Sample Report, 1952" and "234-5 Analytical Laboratory Daily Sample Report, Spectro-Chemical Analysis, 1952".

b. Tabular Summary

The reader's attention is invited to the following table of results; these findings will be discussed in more detail in the subsequent paragraphs.

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Impurities	Average Amount	S. D. Of Observed Amount	S. D. Of True Amount	No. of Measurements Upon Which Results Are Based
"70-58"	10,130.82 ppm.	862.08 ppm.	409.19 ppm.	292 (2 on ea. of 146 items)
Spectro-Chemical	906.72	406.31	295.01	318 (2 on ea. of 159 items)
Carbon	351.29	128.42	$\leq$128.42	151
Fluorine	3.47	2.79	$\leq$ 2.79	153
Boron	0.96	0.69	$\leq$ 0.69	153
Total Amount	11,393.26 ppm.	962 ppm.	$\geq$504; $\leq$521	---

c. Discussion of Results

(1) The total amount of these impurities was found to average 11,393 parts per million (or 1.14 percent). However, the impurities vary from one shape to another, as reflected in the standard deviations of observed amounts. Yet, not all of this variation is due to fluctuation in the actual impurities; some can be traced to random errors in their measurement. Fortunately, duplicate measurements were available for the two major impurities, thus permitting an evaluation of measurement error as reported in paragraph (2.d). For these two impurities, the standard deviations of actual amounts were obtained by subtracting the variances of measurement errors from the variances of observed amounts. Finally, the standard deviation of the actual total amounts of impurities was estimated to be between 504 and 521 ppm., for which 510 will be used here as a reasonable compromise. However, it is rather immaterial which of these three values is used since, correct to the two significant digits recommended for use, the same precision figures follow from all three values.

(2) Since it is desired to use the same average to correct all shapes for impurities, the precision of this procedure will depend upon the standard deviation of 510 ppm. as an index of the variation of actual total impurities in individual shapes. Thus, content of a single shape may be stated as:

$$\text{Impurities} = G\,(0.0114 \pm 0.0009996)$$
$$\text{Plutonium} = G\,(0.9886 \pm 0.0009996)$$
$$= G\,(0.9886)\;(1.00000 \pm 0.00101)$$

where G = gross weight before correction

and where 0.0114 represents the 1.14 percent average total impurities found by the present study,

0.9886 is the correction factor indicated by the present study,

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0.0009996 represents the precision expressed as 999.6 ppm.

0.00101 represents the precision as 0.101 percent of the estimated net weight.

(All precisions stated on 95 percent level.)

Also, the average or total content of n shapes may be stated as:

$$\text{Impurities} = G\left(0.0114 \pm \frac{0.0009996}{\sqrt{n}}\right)$$

$$\text{Plutonium} = G\left(0.9886 \pm \frac{0.0009996}{\sqrt{n}}\right)$$

$$= G\,(0.9886)\left(1.00000 \pm \frac{0.00101}{\sqrt{n}}\right)$$

where G = average or total gross weight.

In other words, the precision in units of weight may be obtained by multiplying G by $(0.0009996/\sqrt{n})$ or by multiplying the corrected or net weight by $(0.00101/\sqrt{n})$. However, for either multiplication a factor of $(0.0010/\sqrt{n})$ should be used since it carries as many digits as is justified by the underlying computations.

The results of this study are presented in brief form in paragraph (2).

5. RE-EXAMINATION

It is recommended that provision be made for periodic re-examination of the correction factor and the precision applicable to the plutonium content of shapes.

N. D. Peterson
N. D. Peterson
STATISTICS UNIT

ND Peterson:dc